

COMPARING RATES of CHANGE in LINEAR & QUADRATIC FUNCTIONS

Definitions:

DEGREE of a polynomial: For a polynomial with a single variable, the **degree** is the value of the highest exponent of that variable.

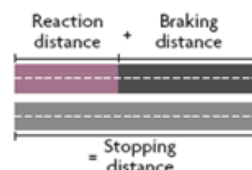
Eg. The polynomial $5x^3 - 4x^2 + 7x - 8$ has a **degree of 3**.

FUNCTION NOTATION: $f(x)$ is called function notation and represents the value of the dependent variable for a given value of x . ($f(x)$ is read "f of x"; other letters may be used.)

$$y = 2x + 3 \quad f(x) = 2x + 3$$

Example #1: Comparing Linear & Quadratic Functions

Stopping distance is the distance a car travels from the time the driver decides to apply the brakes to the time the car stops.



A) Comparing speed and reaction distance:

Speed (m/s)	Reaction Distance (m)	First Differences
0.00	0.00	
5.56	8.33	
11.11	16.67	
16.67	25.00	
22.22	33.33	
27.78	41.67	

\therefore the first differences are the same

\therefore linear function

Equation:

$$y = mx + b$$

$$(0, 0) \quad (5.56, 8.33)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8.33 - 0}{5.56 - 0} = \frac{8.33}{5.56} = 1.50$$

Domain: $\{x \in \mathbb{R} | x \geq 0\}$

Range: $\{y \in \mathbb{R} | y \geq 0\}$

$$y = 1.50x + 0$$

$$y = 1.50x + 0$$

B) Comparing speed and braking distance:

Speed (m/s)	Braking Distance (m)	First Differences	Second Differences
0.00	0.00		
5.56	1.77		
11.11	7.10		
16.67	15.96		
22.22	28.38		
27.78	44.35		

\therefore the 2nd differences are the same

it is a quadratic function

Equation:

$$y = ax^2 + bx + c$$

$$y = 0.0575x^2$$

$$y = g(x) = 0.0575x^2$$

$$(Degree is 2)$$

Domain: $\{x \in \mathbb{R} | x \geq 0\}$

Range: $\{y \in \mathbb{R} | y \geq 0\}$

$$y = 0.0575x^2$$

Example #2: Representing Distance Travelled as a Function of Time

A migrating butterfly travels about 128 km each day. The distance it travels, $g(t)$ in kilometers, is a function of time, t in days.

- a) Write an equation using function notation to represent the distance a butterfly travels in t days.

$$\underline{g(t) = 128t}$$

- b) State the degree of this function. Is it linear or quadratic?

The degree is 1. Therefore the function is linear.

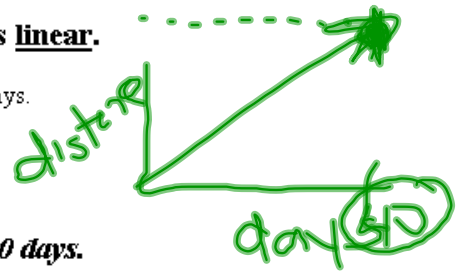
- c) Use the function to calculate the distance the butterfly travels in 20 days.

$$t = 20$$

$$g(t) = 128t$$

$$g(20) = 128(20) = 2560 \text{ km}$$

Therefore, the butterfly travels 2560 km in 20 days.



- d) What are the domain and range of this function in set notation? (Assume the butterfly lives 40 days, on average.)

$$\text{Domain: } D = \{t \in \mathbb{R} \mid 0 \leq t \leq 40\}$$

$$\text{Range: } R = \{g(t) \in \mathbb{R} \mid 0 \leq g(t) \leq 5120\} \quad \text{since } g(40) = 128(40) = 5120$$

Note: neither distance nor time can be negative.

Example #3: Connecting Degree to Type of Function

State the **degree** of each function and identify which functions are **linear** and which are **quadratic**.

a) $k(x) = 3x(x+1)$

$$k(x) = 3x^2 + 3x$$

degree: 2

Quadratic -

b) $m(x) = (x+2)^2 - x^2$

$$= (x+2)(x+2) - x^2$$

$$= x^2 + 2x + 2x + 4 - x^2$$

$$= 4x + 4$$

degree: 1

Linear

SUMMARY:

Linear functions have constant first differences, a degree of 1, graphs that are linear

Quadratic functions have constant 2nd differences, a degree of 2, graphs that are

quadratic/parabolic

